An Introduction to Discrete Mathematics

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Seeing there is nothing (right well beloved Students in the Mathematickes) that is so troublesome to Mathematical practise, nor that doth more molest and hinder Calcuations, then the Multiplications, Divisions, square and cubical Extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my minde, by what certaine and ready Art I might remove those hindrances. (John Napier, 1614)

To Donna

Preface

Let me begin by thanking the many professors and students who have used the second edition of my book *An Introduction to Discrete Mathematics*" since it first appeared in 1989.

For this edition, I have included several new topics, primarily in the chapters on combinatorics and graph theory, including a new chapter on advanced counting techniques. Let me mention just a few new items.

1) More proofs in the chapter on graph theory.
2) A discussion of Stirling numbers and partitions and placing distinguishable balls into indistinguishable boxes.
3) A discussion of the remarkable Catalan numbers, based on my book *An Introduction to Catalan Numbers*, published by Birkhäuser.
4) A discussion of partitions of an integer and distributing indistinguishable balls into indistinguishable boxes.

In the mid 1980's, a considerable interest arose in the subject of discrete mathematics as a result of the upsurge in the number of computer science majors. Although this interest has waned considerably as the number of computer science majors—many of whom thought that computer science was synonymous with computer programming—has waned, this subject is still of critical importance to both computer science majors and in my opinion, even more so to mathematics majors.

I hope this book will fill the needs of both types of students. The book is intended to be an elementary introduction to certain topics in discrete mathematics and not an encyclopedic treatment of the subject. It's prerequisites are deliberately kept to a minimum, requiring no college-level mathematics. Nevertheless, the books starts out at a modest level, with material that may already be familiar to the reader, but does increase in level as the story unfolds. One of my goals is to introduce the reader to the "mathematical" way of thinking, that is, to the ideas of a definition, a theorem, and a proof.

Let me briefly discuss the contents of the book.

1) Chapter I covers the basic prerequisites for the course.
2) Chapter 2 is devoted to elementary logic. There are two main goals here. One is to discuss the concept of logical equivalence, so that the relationships between a statement, its converse, inverse, and contrapositive can be made clear. The other goal is to discuss the relationship between logic and elementary circuit design. This includes a discussion of Boolean functions. Also, Karnaugh maps are discussed in the last section of the chapter.
3) In Chapter 3 we discuss relations on sets, including equivalence relations and partially ordered sets. Topological sorting is included as an application. The last section of the chapter introduces the student to the concept of a morphism (of order).

4) Chapters 4 contains a fairly complete introduction to elementary combinatorics: permutations, combinations and binomial coefficients, multinomial coefficients, multiset coefficients and so on.
5) Chapter 5 is devoted to the important topic of recurrence relations.
6) Chapter 6 contains a thorough discussion of the principle of inclusion-exclusion, along with an introduction to probability.
7) Chapter 7 contains a discussion of counting the number of partitions of a set, which involves the Stirling numbers of the second kind and counting permutations with a given number of cycles in their cyclic decomposition, which involves the Stirling numbers of the first kind. I also discuss the Catalan numbers in this chapter.
8) Finally, Chapter 8 is devoted to graph theory, where we discuss both directed and undirected graphs. Some of the more applied topics included are the depth first search, binary search trees, Huffman codes, the minimal spanning tree problem, and the shortest path problem. Several algorithms are discussed, but they are given in English rather than in pseudo code; thus, no knowledge of pseudo code is necessary. The final section is devoted to finite state machines.

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